TRANSITION FROM KINETIC TO GASDYNAMIC PLASMA RETENTION IN A PLUGOTRON

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1. Introduction. In [1, 2], there is a discussion of the scope for using a trap with magnetic plugs to retain a fusion plasma in the collisional state, when the length of the system L and the plug ratio R are so large that the ion mean free path λ/R in relation to the scattering through an angle of the order of the loss cone satisfies the inequality

$$\lambda/R \ll L. \tag{1.1}$$

That case is directly contrary to the conditions of plasma retention in a classical machine working with infrequent collisions ($\lambda/R \gg L$). We retain the terminology of [1, 2] and subsequently call this system a gasdynamic trap GDT.*

In [1, 2], a study was made of the scope for producing a stationary fusion reactor with neutral injection into a region of homogeneous magnetic field. The optimum parameters were found for various values of the amplification coefficient Q. It was assumed in the calculations that the ion distribution is Maxwellian. In fact, it may differ substantially from Maxwellian at high energies.† The non-Maxwellian form of the tail in the distribution is associated with the presence of a source at high energies, and also with the feature that condition (1.1) may be violated for high-energy ions because of the energy dependence of the collisional cross section.

Some analytical calcultions were performed in [3] that incorporated the contribution from the ions in the tail to the nuclear energy production on the assumption that the number of ions δn in the high-energy part of the spectrum is small by comparison with the number n belonging to the low-energy Maxwellian part. On varying $\delta n/n$ it was found that this ratio ceases to be small in forms optimal from the reactor viewpoint. As the accuracy of analytical calculations in that case becomes inadequate, the Fokker–Planck kinetic equation FPE has to be solved numerically to derive the ion distribution and then to optimize the parameters of the GDT reactor.

Numerical solutions to the FPE for a classical collisionless state are familiar [4-7]. In the rectangular magnetic well approximation, the distribution is independent of the coordinate z. It is determined by the stationary solution to the two-dimensional FPE containing the variables v and θ together with a boundary condition corresponding to the distribution becoming zero at angles θ lying within the loss cone.

In the opposite limiting case, many of the ions are retained under gasdynamic conditions, and the loss cone is filled, and the distribution is close to Maxwellian. The dependence on the z coordinate appears in the distribution via macroscopic parameters such as the density, temperature, and bulk velocity, and it can be found from the MHD equations. The Maxwellian form is distorted on scales of λ/R near the mirrors (an unfilled cone occurs for particles moving towards the center of the trap). This scale is small by comparison with L for most of the ions by virtue of (1.1). However, the mean free path increases with the energy of the colliding particles, so it becomes large in the tail of the distribution. For the small number of tail particles whose mean free paths are $\lambda(v) \geqslant LR$, the deviations in the distribution from Maxwellian extend to the entire trap. For these particles, the retention conditions are characteristic of a classical machine; they will subsequently be called kinetic.

This shows that the case is transitional between the gasdynamic and kinetic ones when $\lambda(v_{Ti})/R \simeq L$ for the main body of ions. Then the dependence of the distribution shape on the

*The name is given because the ion velocity distribution is close to Maxwellian if they were in the trap apart from the region around mirrors, while the longitudinal flow is described by the gasdynamic equations.

†In [2] this feature was incorporated approximately via the numerical factor ξ = 1.5-2.

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z coordinate is substantial. It is also necessary to solve the problem with respect to the z coordinate to find this relationship, as well as solving the FPE in velocity space.

Here we consider the numerical integration of the FPE, which enables one to trace the transition to the stationary solution for the three-dimensional distribution f(t, z, v, θ). This requires a long run time with an ordinary computer, so in addition to the program for the BÉSM-6 we wrote a second form of program enabling one to solve the problem on a PS-2000 computer with the use of parallel processors. As the main task at this stage was to check the program operation, we used the maximally simplified physical formulation of the problem, in which we envisaged the moderate value R = 2 and did not incorporate the collisions of the ions with electrons or the ambipolar potential in the mirror, and correspondingly the hyperbolic surface on which the boundary condition should be imposed was replaced by the surface of a cone.

2. Formulation. In accordance with the above, we assume that the distribution is dependent on the variables v, θ, z, and t, where z is the coordinate along the lines of force, v is the modulus of the ion velocity, and θ is the angle between the magnetic field and the particle velocity. For definiteness we assume that the point z = 0 corresponds to the center of the trap, while the points z = ±L/2 correspond to the mirrors. We assume that the lines of force in the magnetic field are symmetrical about the center, so the problem need be considered only over the interval $0 \leqslant z \leqslant L/2$.

We also assume that the scale in the region of transition from the homogeneous magnetic field in the trap to the mirror is small by comparison with the length of the system, and that the ions move through this region without collision. In that case, we can assume that the lines of force form a rectangular magnetic well having point plugs, and we incorporate the presence of the latter by means of certain boundary conditions imposed on the distribution at z = L/2. We formulate these boundary conditions.

At points adjoining the central part of the trap and directly adjoining the mirror (z = L/2 − 0), we distinguish particles whose velocity vectors lie within and outside the loss cone. The shut-off particles lying outside the loss cone are reflected from the magnetic potential, and correspondingly their distribution should be symmetrical in θ about the point θ = π/2. The ions lying within the half of the cone facing towards the mirror are not reflected and escape from the trap, so in the second half of the cone, which faces backward, the distribution is equal to zero. Then the boundary conditions at z = L/2 are written as

$$F(\pi-\theta,\ v,\ L/2) = F(\theta,\ v,\ L/2),\quad \theta_0 \leqslant \theta \leqslant \pi/2,$$
$$F(\theta,\ v,\ L/2) = 0,\quad \pi \geqslant \theta \geqslant \pi-\theta_0.$$

The boundary condition at z = 0 follows from the symmetry of the distribution about the center:

$$F(\theta,\ v,\ 0) = F(\pi-\theta,\ v,\ 0),\quad \partial F/\partial z|_{z=0} = 0.$$

The third and fourth boundary conditions are that the particle fluxes are zero in velocity space at the point v = 0 and on the lines θ = 0, π.

We write the Fokker–Planck equations incorporating the collisions of ions with ions as

$$\frac{\partial f}{\partial t} + v\cos\theta\frac{\partial f}{\partial z} = \frac{1}{v^2}\left\{\frac{\partial}{\partial v}\left(\frac{\partial W}{\partial v}\right) + \frac{1}{\sin\theta}\frac{\partial P}{\partial\theta}\right\} + \frac{\dot N(z)}{2\pi v^2}\delta(v-\bar v)\,\delta\left(\theta-\frac{\pi}{2}\right),$$
$$W = \frac{1}{2v}\frac{\partial}{\partial v}\left(\frac{\partial^2 G}{\partial v^2}v^3 f\right) - v\widetilde H,\quad P = A\frac{\partial f}{\partial\theta} + B\frac{\partial f}{\partial v} + Cf,$$
$$A = \frac{1}{2}\sin\theta\left(\frac{1}{v}\frac{\partial G}{\partial v} + \frac{1}{v^2}\frac{\partial^2 G}{\partial\theta^2}\right),\quad B = v\sin\theta\frac{\partial^2}{\partial v\partial\theta}\left(\frac{G}{v}\right),$$
$$C = \frac{1}{2}\left(\frac{\partial B}{\partial v} - \sin\theta\frac{\partial H}{\partial\theta}\right), \tag{2.1}$$
$$G = \Gamma\int f(\mathbf v')\,|\mathbf v-\mathbf v'|\,d\mathbf v',\quad \Gamma = \frac{4\pi e^4\Lambda}{m_i^2},$$
$$H = 2\Gamma\int f(\mathbf v')\,|\mathbf v-\mathbf v'|^{-1}d\mathbf v',$$
$$\widetilde H = \Gamma\int(\mathbf v^2-\mathbf v'^2)\,|\mathbf v-\mathbf v'|^{-3}f(\mathbf v')\,d\mathbf v'.$$

This form is convenient for constructing a conservative difference scheme that conserves the energy and the number of particles [8, 9]. Here by $\dot{N}$ we denote the number of ions generated in unit volumes in unit time, while $\bar{v}$ is the velocity of these particles. It is assumed that the injection is performed at a single energy perpendicular to the magnetic field. We note that in (2.1) we have omitted the term resulting from the component of the electric field along the lines of force of the magnetic field in order to simplify the calculations. With a Boltzmann electron distribution, this field is expressed in terms of the gradient in the electron pressure, so this imposes a constraint on the electron temperature $T_e \ll T_i$.

We reduce (2.1) to dimensionless form. For this we introduce the dimensionless distribution F and the variables u, τ, and x in accordance with

$$f = f_0 F,\ v = v_0 u,\ t = t_0 \tau,\ z = xL/2.$$

We substitute these expressions into (2.1) to get

$$\frac{f_0}{t_0}\left(\frac{\partial F}{\partial \tau} + \frac{2v_0 t_0}{L} u \cos\theta \frac{\partial F}{\partial x}\right) = \Gamma f_0^2 \frac{1}{u^2}\left\{\frac{\partial W}{\partial u} + \frac{1}{\sin\theta}\frac{\partial P}{\partial \theta}\right\} + \frac{\dot{N}}{2\pi v_0^2 u^2}\delta\left(u - \frac{\bar{v}}{v_0}\right)\delta\left(\theta - \frac{\pi}{2}\right). \tag{2.2}$$

The parameters so far undetermined that were used in rendering the equation dimensionless are selected from the condition that the coefficients to the terms in (2.2) become one. We then have the system of equations

$$t_0 = L/(2v_0),\quad f_0 = 2v_0/(\Gamma L),\quad \dot{N}\Gamma L^2/(4v_0^5) = 1,$$

from which one can find the values of the parameters f_0, v_0, t_0. The Fokker–Planck equation takes the following form in the new variables:

$$\frac{\partial F}{\partial \tau} + u\cos\theta \frac{\partial F}{\partial x} = \frac{1}{u^2}\left[\frac{\partial W}{\partial u} + \frac{1}{\sin\theta}\frac{\partial P}{\partial \theta}\right] + \frac{\delta(u - \bar{u})\,\delta\left(\theta - \frac{\pi}{2}\right)}{2\pi u^2}, \tag{2.3}$$

where $\bar{u} = \bar{v}/v_0$.

To solve (2.3) numerically, we introduce a dimensional difference net on x, u, θ, τ ($0 \leqslant x \leqslant 1$, $0 \leqslant \theta \leqslant \pi$, $0 \leqslant u \leqslant u_0$, $\tau \geqslant 0$):

$$x_k = k\delta x,\ k = 0, 1, \ldots, K;\ u_i = i\delta x,\ i = 0, 1, \ldots, I;$$
$$\theta_j = j\delta\theta,\ j = 0, 1, \ldots, J;\ \tau_n = n\delta\tau,\ n = 0, 1, \ldots$$

On introducing the corresponding grid functions, we readily construct a difference scheme that conserves the number of particles and the energy:

$$\frac{(F^{n+1} - F^n)_{i,j,k}}{\delta\tau} + u_i \cos\theta_j\left\{\frac{(F^n_{k+1} - F^n_k)_{i,j}}{2\delta x}[1 - \mathrm{sign}(\cos\theta_j)] + \right.$$
$$\left. + \frac{(F^n_k - F^n_{k-1})_{i,j}}{2\delta x}[1 + \mathrm{sign}(\cos\theta_j)]\right\} = \frac{1}{u_i^2}\left\{\left(\frac{W^n_{i+1} - W^n_i}{\delta x^2} - \frac{W^n_i - W^n_{i-1}}{\delta x^2 u^2_{i-1/2}}\right)_{j,k} + \right. \tag{2.4}$$
$$\left. + \frac{(P^n_{j+1/2} - P^n_{j-1/2})_{i,k}}{\delta\theta}\right\} + S_{i,j,k}$$

(see [8, 9] for the form of the grid functions $W^n_{i,j}$ and $P^n_{i,j+1/2}$). Here $S_{i,j,k}$ is the difference approximation for the source in (2.2). From (2.4) one can construct an inexplicit scheme in terms of the variables u and θ. However, the scheme remains explicit with respect to x, which imposes a constraint on the τ step, $\delta\tau < \delta x/u_0$.

The calculations were performed with BÉSM-6 and PS-2000 computers. The results given below were obtained with the BÉSM-6. The numbers of points in the variables u and θ were correspondingly 51 and 25, while the number in x was varied from 8 to 32. The integration region in u was of the order of three times the thermal velocities of the ions. We also used a stabilizing-correction scheme [10].

3. Transitions from Kinetic Ion Retention to Gasdynamic. When the variables F, u, τ, and x have been introduced, the only dimensionless parameter in (2.3) and in the boundary conditions is $\bar{u}$ (apart from the ratio R), and this governs the distribution parametrically, and

hence governs the values for the density, temperature,* and other macroscopic parameters. If R is fixed and is of the order of one, all the coefficients in (2.3) and in the boundary conditions are quantities of the order of one, and it is evident that the transition from the kinetic state to the gasdynamic one should occur at $\bar{u} \sim 1$. Here the region $\bar{u} < 1$ corresponds to the gasdynamic state and $\bar{u} > 1$ to the kinetic one. We give below numerical calculations illustrating the transition and emergence on the corresponding asymptotes at large and small values of $\bar{u}$.

Consider the ratio of the flux of particles q leaving the trap in unit time to the asymptotic values of the corresponding fluxes in the limiting gasdynamic and kinetic cases. In pure gasdynamic retention, the ion distribution around the mirror is close to Maxwellian with the half-cone cut out [2]. We integrate this with weight v and multiply the result by the area of cross section S in the central part of the trap to get an expression for the particle flux through the mirror:

$$q_r = \frac{nS}{\sqrt{2\pi}R}\sqrt{\frac{T}{m_i}},$$

where n and T are the density and temperature at the edge of the trap. On the other hand, in this stationary problem the number of ions emerging from the trap in unit time is always constant at $q = LSN/2$. We calculate n and T from the values of the distribution obtained from the numerical computation:

$$n = f_0 v_0^3 \int F(\mathbf{u}, 1)\, d\mathbf{u} = f_0 v_0^3 I_0(1); \tag{3.1}$$

$$T = f_0 \frac{2v_0^5}{3n} \int \frac{m_i u^2}{2} F(\mathbf{u}, 1)\, d\mathbf{u} = \frac{1}{3} m_i v_0^2 \frac{I_2(1)}{I_0(1)}, \tag{3.2}$$

to get the expression

$$\frac{q_r}{q} = \frac{\sqrt{I_0(1)\, I_2(1)}}{R\sqrt{6\pi}}.$$

Curve 1 in Fig. 1 shows this quantity as a function of $\bar{u}$. It is evident that at small values ($\bar{u} < 1$), the ratio q_r/q ceases to be dependent on $\bar{u}$ and tends to one.

We now consider the asymptote relating to the kinetic condition. We know [11] that in this case the ion lifetimes in the system can be estimated from

$$\tau \simeq \tau_{ii}, \tag{3.3}$$

where $\tau_{ii} = T^{3/2}\sqrt{m_i}/(\sqrt{2}\Lambda e^4 n)$.

We substitute for n and T from (3.1) and (3.2) to get

$$q_k/q = I_0^{7/2}/I_2^{3/2}.$$

Curve 2 in Fig. 1 shows the dependence of this ratio on $\bar{u}$, and it is evident that for $\bar{u} > 1$ the ion lifetime is closely described by (3.3).

These curves show that numerical calculations on the distribution give reasonable values in the sense of emergence on the corresponding asymptotes at large and small values of $\bar{u}$. The distributions enable one to construct the profiles for the density n(x) and the temperature T(x) for various values of the parameter $\bar{u}$ (Figs. 2 and 3, where curves 1-4 correspond to $\bar{u}_1 = 1.6$, $\bar{u}_2 = 2.1$, $\bar{u}_3 = 2.82$, $\bar{u}_4 = 3.67$). It is of interest to examine how the gasdynamic parameter RL/λ varies with $\bar{u}$. If we determine the ion mean free path in the form†

$$\lambda = 1/n\sigma, \text{ where } \sigma = e^4\Lambda/T^2,$$

then from (3.1) and (3.2) we get

$$\frac{RL}{\lambda} = \frac{9}{2\pi} R \frac{I_0^3}{I_2^2}.$$

*As the distribution may differ from Maxwellian, the temperature is introduced via $(3/2)nT = E$, where E is the energy in unit volume. The energy of the coherent motion of the plasma as a whole for $R \gg 1$ is small by comparison with the thermal energy ($\sim nT/R^2$).

†We note that the mean free path is close to that introduced in [12] with this definition.

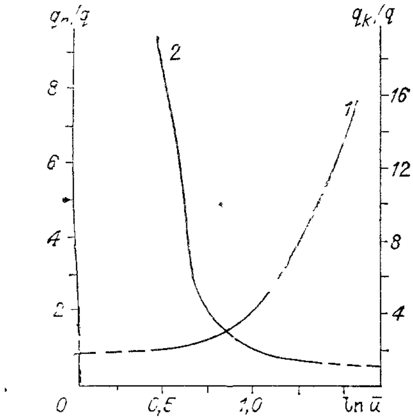

Fig. 1

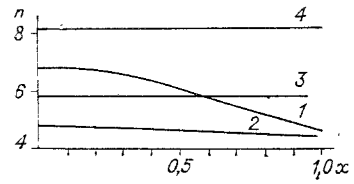

Fig. 2

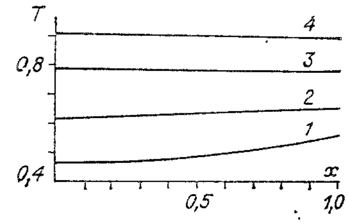

Fig. 3

Figure 4 shows the graph for the right-hand side of this as a function of $\bar{u}$. Comparison with Fig. 1 shows that the transition from the gasdynamic state to the kinetic one occurs at $RL/\lambda \simeq$ 1-2.

An important characteristic of a magnetic trap is the ratio of the ion temperature to the energy introduced into the system per ion. The amounts of energy entering the system and leaving it per ion are $m_i\bar{v}^2/2$. The ratio of the ion temperature of (3.2) to this quantity is

$$\eta = \frac{2T}{m\bar{v}^2} = \frac{2}{3}\frac{1}{\bar{u}^2}\frac{I_2}{I_0}.$$

Figure 5 illustrates the variation of η with $\bar{u}$ (η = 1/2 in the gasdynamic state and $\eta \simeq 1$ in the kinetic one).

We note that there is very restricted application for ordinary difference schemes to numerical solution of the Fokker–Planck equation, i.e., schemes that do not conserve the energy and the number of particles simultaneously, because there are fictitious losses connected with the use of nonconservative schemes, and this disrupts the balance between the sinks and sources for the particles and for the energy from the real sources, and it is necessary to reduce the step in the difference net in an unjustified fashion in order to reduce these losses [13]. This feature is very important in our case, since the calculations in a gasdynamic trap involve solving the three-dimensional Fokker–Planck equation, which requires long run times and large memory volumes.

We show for pure gasdynamic retention that there are constraints on the step in v that would arise from using an ordinary difference scheme that does not conserve the energy in solving the Fokker–Planck equation. We write the equations for the balance between the incoming particle energy and the longitudinal losses from the GDT:

$$LSN/2 = q_{rs} \quad \frac{m\bar{v}^2}{2} q_r = 2Tq_r + A\frac{mh_v^2 n^2 SL}{T^{3/2}}. \tag{3.4}$$

Here the last term in the second equation describes the fictitious energy sink, while h_v is the v step in the difference scheme and A is a numerical coefficient. It is evident from (3.4) that the Fokker–Planck equation does not have any stationary numerical solutions at all for $h_v/\bar{v} > B/\sqrt{LR/\lambda}$ (B is a numerical coefficient). However, when one uses conservative schemes (schemes that conserve the energy and number of particles), these deficiencies are absent.

In conclusion we note that the speed and executive store of the BÉSM-6 are not sufficient to produce physically interesting results for a GDT with R = 20-50. In [14] there is a discussion of the scope for solving this problem with a multiprocessor PS-2000 computer (see [15] for a description of the PS-2000).* The calculations on one form of the problem (GDT with R = 2) took the eight-processor form of the PS-2000 3.75 min, while the BÉSM-6 with a program in the ALPHA-6 language and compilation optimization required 51.2 min without allowance for the processor time consumed in data exchange with the disks. The actual processor time in the BÉSM-6 was twice this.

*In these calculations, the number of points in x was 8, while those in u and θ were correspondingly 41 and 25.

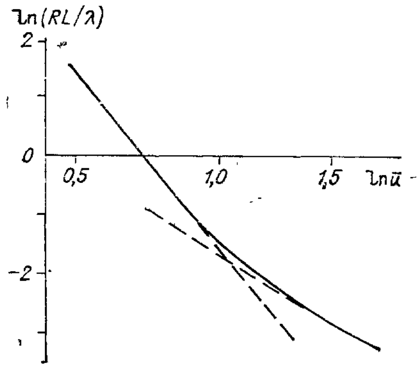

Fig. 4

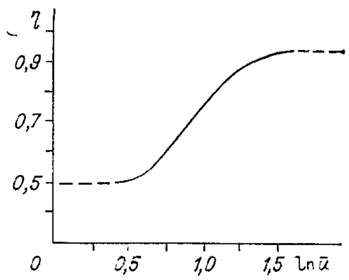

Fig. 5

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